IMOS Thermal Modeling

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OVERVIEW

- IMOS thermal module objectives
- IMOS thermal model
- IMOS internal solver
- Future work

Objectives

- Try to please everybody (accomplish integrated modeling goals + retain standard thermal modeling procedures)... *Work in Progress*
- Develop Matlab based routines for defining/solving thermal models
  - Compatibility with structural model (automatic generation of conduction conductors)
  - Compatibility with thermal modeling methods and programs
  - Flexibility to allow user to create/modify elements of model (e.g. add radiator conductors, one–way nodes, etc.)
  - Solve steady state and transient problems
- Not a current objective to develop radiation conductors (although ability to generate input files for TRASYS from f.e. mesh is a goal)

Status

- Automatic conversion of beam and plate elements from restricted finite element geometry
- User defined thermal nodes and conductors can be added
- Temperature varying material conductance properties can be supplied by tabular input
- Steady state solver tested (includes radiation, temperature varying linear conductors)

Modeling — The IMOS Plate Element

- The IMOS plate element is a 2–D *triangular* element with no interior angle greater than 90°
- Why the restrictions?
  - Produces positive conduction elements that:
    - retain network characteristics, compatibility with standard methods
    - lead to "diagonally dominant" systems (functional iteration methods need this condition!)
  - Conductances can be generated from integral volume methods, i.e. each node is associated with a volume:
    - finite difference and finite element interpretation
    - leaves room for adding radiation conductors

Integral Volume Approach

Consider heat equation

$$\nabla \cdot (k\nabla u) = f,$$

$u =$ temperature.
$k =$ thermal conductivity,
$f =$ heat generation

For region V with smooth boundary ∂V

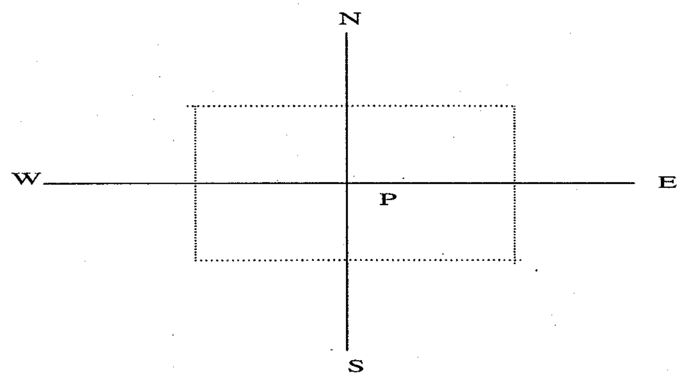
$$\int_V \nabla \cdot (k\nabla u) dv = \int_V f dv.$$

Integral volume approach derived from Green's theorem:

$$\int_V \nabla \cdot (k\nabla u) dv = \int_{\partial V} k\nabla u \cdot n d\sigma$$

$n =$ outward normal vector
$d\sigma =$ surface differential.

Example: Application to a finite difference approximation on uniform rectangular mesh



Control Volume in Uniform Mesh

Approximate $\nabla u \cdot n$ on east boundary by centered difference

$$\nabla u \cdot n \approx \frac{u(E) - u(P)}{h},$$

Analogous approximations on north, west south boundaries lead to

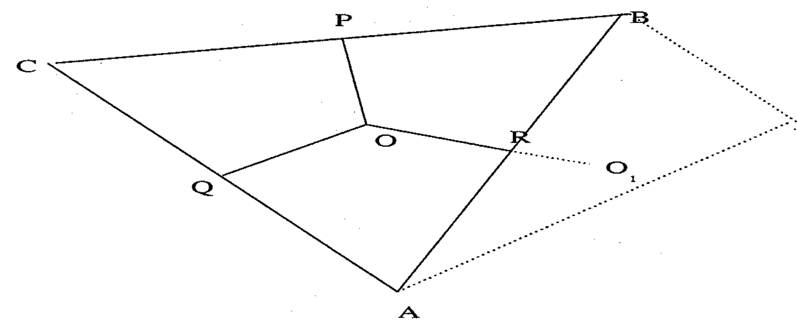
$$\int_{\partial V} k \nabla u \cdot n d\sigma \approx kt[u(E) + u(N) + u(W) + u(S) - 4u(P)], \quad t = \text{plate thickness}$$

Conductances between P and adjacent points are:

$$C_{PE} = C_{PN} = C_{PW} = C_{PS} = \frac{kA}{l},$$

where $A = th$ cross–sectional area and $l = h$ (distance between nodes)

Conductances for the Triangular Element



Control Volume Associated with Triangular Element

Q, R, and P = midpoints of segments AC, AB, and BC, respectively,
O = intersection point of perpendicular bisectors of AC, AB, and BC
V_0 = region bounded by the polygon $AQOR$.
V_0 = control volume assoc. with node A

Conductance Formula:

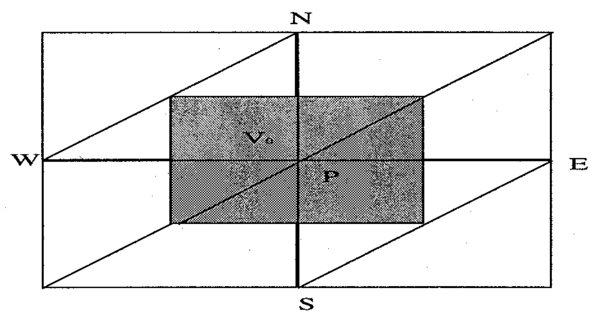
$$C_{AB} = kt \cot C/2, \quad t = \text{plate thickness}$$

Capacitance Formula:

$$V_0 = \frac{t(|AC|^2 \cot B + |AB|^2) \cot C}{8}$$

Plate Element Attributes

- Equivalent to 5–point Laplacian formula on regular mesh



- All conductors are positive when $\angle C \leq 90°$

- *Equivalent* to triangular plate element with picewise linear polynomial on irregular mesh

Steady State Solution Method

- Newton method with linesearch
- Takes advantage of Matlab matrix sparsity routines
- Uses approximation of system Jacobian (exact when there are no temperature varying materials)
- "Globally" convergent (in principle)

Steady State Solution Method

- Steady State Equation

$$F(T) + Q = 0; \quad F(T) = CT + RD(T)$$

T= temperature
C= matrix of linear conductors
R= matrix of radiation conductors
$D(T)$= diag(T_1^4,...,T_N^4)
Q= heat input

- Idea is to minimize $|F(T) + Q|^2$

Can show $F(T) + Q = 0$ has unique solution with $T > 0$...so iteration

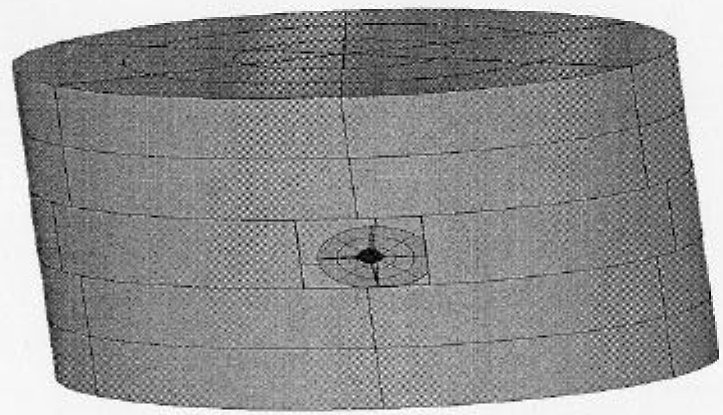
$$T(n+1) = T(n) + s(n)d(n);$$

with

$$d(n) = -F'(T(n))^{-1}F(T(n)), \quad s(n) = \text{steplength parameter}$$

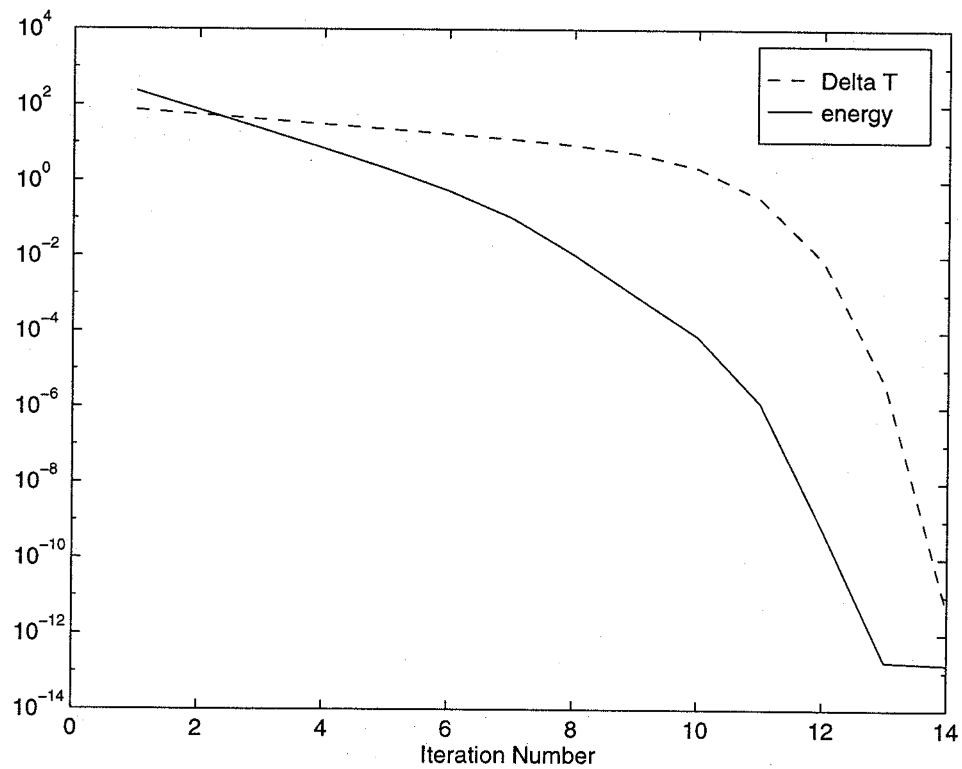
can be shown to be globally convergent (and ultimately quadratic)

Steady State Solver Example



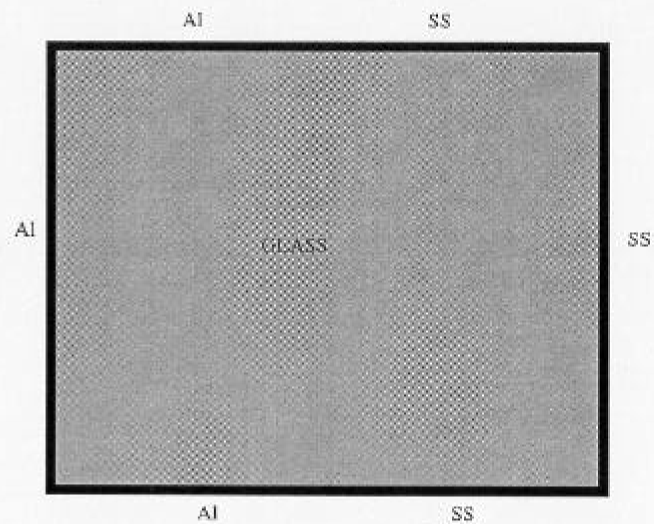
- 362 node system; 8978 radiation elements, 276 linear conductors
- Error tolerance set at 1.0e-8: Sum of energy imbalance + temp. change
- Room temperature initial conditions

Convergence of Steady State Algorithm



Convergence of temperatures and energy balance

Optimization Example



- Objective: control temperature to 310° at certain nodes (*tempset*) by applying heat to another set of nodes (*heatset*) with bounds on heat input
- Model Attributes
  - 20 nodes (3 boundary nodes)
  - Temperature varying materials
  - Radiation conductors

Solution Approach

- Solution approach: Nonlinear least squares problem

T^*= desired temperature vector
tempset= set of nodes to be controlled
heatset= set of nodes at which heat is applied

$$\min_Q |T^* - T(tempset)|^2$$

such that

$$F(T) + Q = 0, \quad 0 \leq Q(heatset) \leq 10, \quad Q(i) = 0, i \notin heatset$$

Nonlinear optimization program *NPSOL* used to obtain solution:

Q^*=[1.313 .3622 .3622 1.313],

T_{opt}=[310.2 310.2 310.4 310.4 310.4 310.2. 310.2 308.0]

- Important Note: "Analytical" gradient of objective functional obtained from F'^{-1}

Future Work

- Transient solver (handle arithmetic nodes)
- TRASYS interface (others)
- One way nodes, code modifications (efficiency,...)
- Other stuff???... optimization package, ID package, submodeling, etc.